

1st Semester Examination, 2022

Time : 3 hours

Full Marks : 60

Answer from **all** the Groups as directed.

*The figures in the right-hand margin
indicate marks.*

*Candidates are required to give their answers
in their own words as far as practicable.*

(MATHEMATICAL PHYSICS-I)

GROUP—A

1. Answer *all* questions :

1 × 8

- (a) For a parabola $y = ax^2 + bx + c$ if a is _____ then the parabola opens upward and if a is _____ then the parabola opens downward.

(Turn Over)

(2)

(b) For the function $y = x^2 - 3x - 4$ the critical point is _____.

(c) A function $f(x, y)$ is called homogeneous of degree n if _____.

(d) The angle between two vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $2\hat{i} - 2\hat{j} + 4\hat{k}$ is _____.

(e) If $z = x^2 - y^2$ $x = r \cos \theta$ $y = r \sin \theta$ then $\frac{\partial z}{\partial r} / y =$ _____.

(f) The value of $\int_0^3 t^3 S(t-5) dt$ is _____.

(g) The vector identity

$$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

(h) The vector field \vec{F} is said to be conservative if \vec{F} can be expressed as _____ of a _____.

GROUP—B

2. Answer any *eight* :

1.5 × 8

(a) Plot the graph of $y = \sin^{-1} x$.

(b) Discuss the continuity of $f(x) = x - |x|$ for all $x \in R$.

(c) Find $\frac{dy}{dx}$ when $x = a(t + \sin t)$
 $y = a \cos t$.

(d) Obtain the Taylor series expansion of $f(x) = e^x$ about $x = 0$.

(e) Obtain the general solution of the equation $y\sqrt{(1+x^2)}dy + x\sqrt{1+y^2}dx = 0$.

(f) Given $u = (y-z)(z-x)(x-y)$ then find

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

(g) Show that

$$\int_{-\infty}^{\infty} \delta(x-a)\delta(x-b)dx = \delta(a-b)$$

(h) If \vec{a} is a const. vector find $\text{grad} (\vec{r} \cdot \vec{a})$.

(i) Find $\text{div curl } \vec{F}$.

(j) If $x = r \cos \theta$ then find $\frac{\partial(r, \theta)}{\partial(x, y)}$.
 $y = r \sin \theta$

GROUP—C

3. Answer any *eight* :

2 × 8

(a) Find $\lim_{n \rightarrow \infty} \frac{n^2 + 3n^3}{5n^3 + 2\sqrt{3+n^6}}$.

(b) Plot the graph of the function
 $y = x^2 - 3x - 4$.

(c) Show that the functions $e^{ax} \sin bx, e^{ax} \cos bx$ are linearly independent with the help of Wronskian.

(d) Solve the equation

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = xe^x \sin x$$

(e) Find $\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k})$

(f) Find the square of the arc length in curvilinear coordinates.

(g) Find the values of p, q and s if $\vec{F} = (4x + 3y + pz)\hat{i} + (qx - y + z)\hat{j} + (2x + 3y + z)\hat{k}$ is irrotational.

(h) Find the unit normal to the surface $xy + yz + zx = 3$ at the point $(1, 1, 1)$.

(i) Given $\vec{A} = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$ what is the projection of $\vec{A} \times \vec{B}$ parallel to $5\hat{i} - \hat{k}$.

- (j) Using Gauss-divergence theorem evaluate the surface integral $(yzdydz + zxdzdx + xydxdy)$ where s is surface of sphere $x^2 + y^2 + z^2 = 9$.

GROUP—D

6 × 4

4. (a) Expand $\tan^{-1} x$ in powers of $(x - 1)$. 3
- (b) Find the approximate value of $(999)^{1/3}$. 3

Or

- (a) Solve the following differential equation $xy' = y + x \sec(y/x)$. 3

(b) $\frac{dy}{dx} = \frac{3 - 2y - 4x}{2x + y - 1}$ 3

5. Discuss the properties of vectors under rotation and also show the invariance of scalar product of two vector under rotation. 6

(7)

Or

- (a) Find the extreme value of $x^3 + 8y^3 + 64z^3$ when $xyz = 1$. 3
- (b) Divide 24 into three parts such that the continued product of the first, square of the second, cube of the third may be minimum. 3
6. Obtain the expression for div, grad and curl in cylindrical coordinates system. 6

Or

Define Dirac Delta function. Discuss the representation of Dirac Delta function as

6

- (a) Limit of Gaussian function.
- (b) Limit of Rectangular function.
7. Define Gradient of a scalar, divergence of a vector and curl of a vector and give their geometrical interpretation. 6

Or

- (a) If R is the projection on XY plane of the surface S then

$$\iint_S \vec{F} \cdot \vec{ds} = \iint_R \vec{F} \cdot \hat{n} \frac{dxdy}{|\hat{n} \cdot \hat{k}|} \quad 3$$

Where \hat{n} is unit normal on S and \hat{k} is the unit normal along z -axis.

- (b) Evaluate $\iint_S \vec{F} \cdot \hat{n} ds$ where

$\vec{F} = \hat{i}z + \hat{j}x - \hat{k}3y^2z$ where $S \Rightarrow$ the surface of cylinder $x^2 + y^2 = 16$ included in the 1st octant between $z = 0, z = 5$. 3

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(MECHANICS)

GROUP—A

1. Answer *all* questions : 1 × 8

(a) Moment of inertia of a sphere about one of its diameters is _____.

(b) The value of coriolis equation is _____.

- (c) ——— is defined as the bending moment required to produce unit radius of curvature in the beam.
- (d) The value of liquid flowing per second through a narrow horizontal tube is given by ———.
- (e) If the total energy E of a particle under attractive inverse square law of forces is +ve then the orbit covered by the particle is ———.
- (f) For a solid sphere the ratio of Gravitational potential at the center and surface is ———.
- (g) The oscillations in the presence of resistive forces are known as ———.
- (h) Circle $x^2 + y^2 = r^2$ in s frame appears to be an ——— in s' which is moving with velocity v with respect to s frame along x -axis.

GROUP—B

2. Answer any *eight* :

1.5 × 8

- (a) Using Routh's rule obtain the moment of inertia of Elliptical Disc about an axis passing through its center and perpendicular to its plane.
- (b) Obtain the magnitude of coriolis force acting on a body of mass 10 gm moving with velocity $(2\hat{i} + 3\hat{j} - 4\hat{k})$ with respect to a rotating frame having angular velocity $(4\hat{i} + 2\hat{j})$.
- (c) State and explain parallel axis theorem.
- (d) For a given material value of ν is 2.4 times that of η . Find the Poisson's Ratio.
- (e) What is Bending moment ? Explain.

(4)

- (f) Give the limitations of Poiseulli's formula.
- (g) Discuss the equivalence of inertial and gravitational mass.
- (h) Discuss some of the physiological effects on astronauts.
- (i) State the postulates of special theory of Relativity.
- (j) Two β -particles travel in opposite directions with velocity $0.9C$. Find the relative velocity.

GROUP—C

3. Answer any *eight* :

2×8

- (a) State and explain Routh's rule for obtaining moment of inertia of symmetrical bodies.
- (b) Discuss the motion of COM system.

- (c) Three particles of mass 2, 4 and 6 gm have the center of mass at point $(2, 2, 2)$. What must be the position of fourth particle of mass 8 gm so that position of center of mass of new system is $(0, 0, 0)$.
- (d) Obtain the work done in elongating a wire.
- (e) A particle of mass 2 kg is at position $(2, -2, 1)$ which moves with velocity $(-\hat{i} + 2\hat{j} - \hat{k})$ m/sec. Find the angular momentum of the particle about x, y and z axis.
- (f) If Gravitational force varies inversely as n th power of distance then show that the time period of a planet in circular orbit of radius r around sun is proportional to $r^{\frac{n+1}{2}}$.

- (g) A particle is in S.H.M along a st. line. It's velocity are v_1 and v_2 when displacements are x_1 and x_2 . Find the frequency of oscillation.
- (h) What is sharpness of Resonance ? Discuss the conditions when the Resonance is sharp and when it is flat.
- (i) Show that the four dimensional volume element is invarient under Lorentz transformation.
- (j) Simultancity is not absolute. Explain.

GROUP—D

Answer *all* questions : 6×4

4. (a) Obtain the kinetic energy in case of rotation and also for the motion involving both rotation and translation. Discuss the rotational motion of an object and hence obtain the expression. 3
- (b) Obtain Euler's equation of motion for a rigid body. 3

(7)

Or

Obtain the moment of inertia of a hollow cylinder : 6

(a) about its axis

(b) about an axis passing through its center and $\perp r$ to its own axis.

5. What are Gravity waves and ripples ? Explain. Obtain the expression for velocity of both and hence obtain the condition when the waves are called Gravity waves and when the waves are called ripples. 6

Or

What is a cantilever ? Obtain the expression for depression in case of a single cantilever. 6

6. State and prove Kepler's laws of planetary motion. 6

Or

Or

Obtain the expression for the gravitational potential due to a spherical shell 6

(a) at an external point P

(b) at an internal point

7. What is doppler effect ? Explain. Derive an expression for relativistic doppler effect. 6

Or

Define S.H.M. Show that in S.H.M the average P.E is equal to average K.E when the average is taken over one complete cycle and each average is equal to half of total energy. Hence represent all the three graphically. 6