1st Semester Examination, 2020

Time : 3 hours

Full Marks : 60

Answer any **one** Group as per your Syllabus.

Answer from all the sections as per direction.

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP—A
(MODEL SYLLABUS)
(MATHEMATICAL PHYSICS-I)

SECTION–A

1. Answer *all* questions : 1×8

(*a*) $y = x$ is a straight line of slope ——.

(*b*) $\lim_{x \to 0} \frac{\sin x}{x} =$ ——.

(*c*) Order of the differential equation $\frac{d^2 y}{dx^2} + 6y = 2x$, is ——.

(*d*) If $U = e^x \cos y$, then $\frac{\partial u}{\partial x} =$ ——.

(*e*) Angle between the two meters $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$ and $\vec{B} = 5\hat{i} + 2\hat{j} + 4\hat{k}$ is ——.

(*f*) For any constant a and dirac delta function $\delta(x)$ $\frac{\delta(-x)}{a} =$ ——.

(*g*) Divergence of a solenoidal vector is ——.

(*h*) $\iint_S \vec{F} \cdot \hat{n} ds =$ ——.

SECTION–B

2. Answer any *eight* of the following : 1.5 × 8

(*a*) Plot the graph $y = x^2$.

(*b*) Find $\lim_{x \to 0} \frac{\left(\sqrt{1-x}\right) - 1}{x}$.

(*c*) Find the general solution of the differential equation $ax\frac{dy}{dx} = by$.

(*d*) Show that the function e^{ax} and e^{-ax} and linearly independent.

(*e*) Check the continuity of the function $f(x, y) = x^2 + 2y$, at (1, 2).

(*f*) Solve, $ydx - xdy = xy^3dy$.

(*g*) Justify the statement – 'If three vectors are co-planar, then the value of the scalar tripple product is zero'.

(*h*) Draw and define cylindrical co-ordinates.

(*i*) Show that grad $(\varphi + \psi) = \text{grad } \varphi + \text{grad } \psi$.

(*j*) Show that $\int_C \vec{r} \cdot d\vec{r} = 0$.

SECTION–C

3. Answer any *eight* of the following : 2 × 8

(*a*) Find $\lim_{x \to 0} \frac{x^2 + 8x}{x}$.

(*b*) Find $\frac{dy}{dx}$, if $x = a(t + \sin t)$, $y = a \cos t$.

(*c*) With a suitable example define a homogeneous differential equation of degree *n*.

(*d*) Solve the differential equation $x\frac{dy}{dx} + y = x^3 + x$.

(*e*) What is wronskian ? What is its application ?

(*f*) Solve the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$.

(*g*) Evaluate $\lim_{\substack{x\to 0 \\ y\to 0}} \frac{xy}{x^2+y^2}$.

(*h*) What is the geometrical interpretation of gradient of a function.

(*i*) For dirac delta function show that

$$f(x)\ \delta(n-a) = f(a)\ \delta(x-a)$$

(*j*) Using Green's theorem show that area of a plane region

$$A = \frac{1}{2}\oint_C (xdy - ydx)$$

SECTION–D

Answer *all* questions :

4. Draw the graph $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ and check the continuity and differentiability of the function. 6

Or

Solve the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = \sin t$. 6

5. If $U = \frac{y}{z} + \frac{z}{x}$, then find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. 6

Or

For unit vectors $\hat{i}, \hat{j}$ & $\hat{k}$ and for any vector $\vec{A}$ find

$$\hat{i}\times\left(\vec{A}\times\hat{i}\right) + \hat{j}\times\left(\vec{A}\times\hat{j}\right) + \hat{k}\times\left(\vec{A}\times\hat{k}\right)$$ 6

6. Find the expression for velocity in spherical polar coordinates. 6

Or

Discuss the properties of Dirac delta function. 6

7. Justify the statement for two scalar function f and g, $\nabla f \times \nabla g$ is solenoidal. 6

Or

State and prove Stoke's theorem. 6

GROUP—B
(OLD SYLLABUS)
(MATHEMATICAL PHYSICS-I)

SECTION–A

1. Answer *all* questions : 2×6

(*a*) Define vector triple product.

(*b*) Find the gradient of scalar function $\phi(xy) = x^2 - y^2$.

(*c*) Explain Lagrange method of undetermined multipliers.

(*d*) Prove that $\delta(-x) = \delta(x)$.

(*e*) Write relation between unit vectors $(\hat{r}, \hat{\theta})$ and $(\hat{i}, \hat{j})$.

(*f*) State Green's theorem.

SECTION–B

Answer *all* questions : 12×4

2. (*a*) Explain scalar triple product with its physical significance and features. 10

(*b*) Distinguish between scalar field and vector field. 2

Or

(*a*) Explain divergence of a vector field and its physical significance with examples. 10

(*b*) Calculate the divergence of the vector

$$\vec{A} = xy\hat{i} + yz\hat{j} + zx\hat{k} \, .$$ 2

3. (*a*) Derive an expression for partial differentiation of vectors. 10

(*b*) Solve the differential equation $\left(1+x^2\right)dy-\left(1+y^2\right)dx=0$. 2

Or

(*a*) Derive relation of Delta function with the step function. 6

(*b*) Prove that $x\cdot\delta(x)=0$. 3

(c) Prove that $f(x)\cdot\delta(x-a)=f(a)\cdot\delta(x-a)$. 3

4. Find the divergence and curl in terms of orthogonal curvilinear coordinates. 6+6

Or

Derive an expression for velocity and acceleration in cylindrical and spherical polar coordinates in three dimension. 6+6

5. State and prove Gauss divergence theorem with its physical significance. 12

Or

Explain line integral, surface integral and volume integral of a vector function. 4+4+4

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GROUP—A
(MODEL SYLLABUS)
(MECHANICS)

SECTION–A

1. Answer *all* questions : 1×8

(*a*) Position vector of C.M in Laboratory frame of a two particle system of mass 1 kg and 2 kg having position vector 1 m and 2 m respectively is ——?

(*b*) Moment of inertia of flywheel can be measured using the principle ——.

(*c*) For an incompressible substance the value of poissons ratio is ——.

(*d*) Gravitational field at an internal point of a solid sphere ——with distance.

(*e*) Bar pendulum is ——pendulum.

(*f*) For a stream line flow Reynold number is ——.

(*g*) Orbital velocity of a sattelite moving very close to earth is ——.

(*h*) Lorentz transformation equations reduces to Galilean transformation equation for ——.

SECTION–B

2. Answer any *eight* of the following : 1.5×8

(*a*) Define radius of gyration.

(*b*) State principle of conservation of angular momentum.

(*c*) Justify the statement Erosion of Reverbanks is due to coriolis force.

(*d*) Why a hollow shaft is stronger than solid one.

(*e*) How the excess pressure in a soap bubble changes by doubling the radius ?

(*f*) Establish a relation between gravitational field and potential.

(*g*) How the time period of revolution of a planet changes by doubling its orbital radius.

(*h*) Explain weightlessness inside a satellite.

(*i*) Define relaxation time in damped oscillation.

(*j*) State Einsteins mass energy relation and explain.

SECTION–C

3. Answer any *eight* questions : 2×8

(*a*) With a suitable example, explain parallel axis theorem of moment of inertia.

(*b*) State and explain Routh rule.

(*c*) Explain Coriolis force with an example.

(*d*) From the basic relation calculate theoretical limiting value of Poissons ratio.

(*e*) Find the work done in elongating a wire.

(*f*) What are the limitations of Poiseuille's formula for viscosity and the corrections.

(*g*) Explain gravity waves with example.

(*h*) Calculate the height of a geo-synchronous orbit.

(*i*) What is Q-factor of forced oscillators ? What is its value in low damped condition ?

(*j*) Calculate the length contraction when a meter rod is moving with 0.8 C with respect to a rest frame.

SECTION–D

Answer all questions :

4. Derive an expression for moment of inertia of a solid cylinder about an axis passing through its axis. 6

Or

Find the expression for coriolis force considering a uniformly rotating frame of reference.

5. Find the expression for bending moment and depression at the free end of a light cantilever. 6

Or

Derive Poiseuille's formula for velocity of flow of a viscous liquid in a narrow tube.

6. Establish Kepler's laws of planetary motion by considering equation of orbit in central force motion. 6

Or

Derive expression for gravitational potential and field at an external point of a solid sphere.

7. From the desired equation, discuss case of overdamped, underdamped and critically damped condition. 6

Or

With Lorentz transformation equations explain time dilation.

GROUP—B

(OLD SYLLABUS)

(MECHANICS)

SECTION–A

1. Answer *all* questions : 2×6

(*a*) Find relation between torque and angular momentum.

(*b*) Define coefficient of elasticity and write its S.I unit.

(*c*) Explain Weightlessness.

(*d*) Explain sharpness of Resonance.

(*e*) Write Lorentz Transformation equations.

(*f*) Distinguish between inertial mass and gravitational mass.

SECTION–B

Answer *all* questions : 12×4

2. (*a*) State parallel and perpendicular axes theorem of Moment of Inertia. 4

(*b*) Derive an expression for Moment of Inertia of a solid sphere rotating about an axis passing through its diameter. 8

Or

Explain the physical significance of fictitious force and prove that coriolis force and centrifugal force are consequence of rotation of frame of reference. 12

3. (*a*) Explain beam, bending moment and cantilever. 6

(*b*) Derive an expression for the depression of the loaded end of a cantilever of circular cross section. 6

Or

Derive an expression for Poiseuille's formula for liquid flowing through a capillary tube. Also discuss about the corrections to Poiseuille's formula. 12

4. Define gravitational field and gravitational potential. Then derive an expression for gravitational potential and field due to a spherical shell at points

(*i*) Outside the surface

(*ii*) On the surface

(*iii*) Inside the shell 12

Or

Explain central force field. Then derive an expression for two body problem and its reduction to one body problem. Also find its solution. 2+10

5. Define simple Harmonic Motion with some examples. Then derive an expression for total energy of a particle executing S.H.M with graphical representation. 4+8

Or

(*a*) Using Lorentz transformation equation derive an expression for Relativistic addition of velocities. 6

(*b*) Explain Energy-Momentum four vector 4

(*c*) Two photons approach each other, what is their relative velocity. 2