Total Pages—7 **SH PHY – 01(MODEL CBCS)**

2019

(1st Semester)

Time : 3 hours

Full Marks : 60

Answer **all** the Parts as per the direction

The figures in the right-hand margin indicate marks

Candidates are required to answer in their own words as far as practicable

(MATHEMATICAL PHYSICS - I)

PART – I

1. Answer *all* questions : 1 × 8

(*a*) The order of the differential equation

$$\frac{d^2x}{dt^2} + w^2x = 0 \text{ is } \_\_\_\_\_.$$

(*b*) The solution of the differential equation

$ydx + xdy = 0$ is _____.

(Turn Over)

(c) $\vec{a}\cdot\left(\vec{a}\times\vec{b}\right) =$ _____ .

(d) What is the volume of the rectangular parallelopiped with sides representing vectors $\vec{a}=(2m)\hat{i}$, $\vec{b}=(3m)\hat{j}$ and $\vec{c}=(1m)\hat{k}$?

(e) Expression for $\vec{\nabla}\cdot\vec{A}$ in orthogonal curvilinear coordinate is _____.

(f) $\delta(ax) =$ _____ $\delta(x)$.

(g) In spherical polar coordinates $\vec{\nabla} =$ _____.

(h) Curl of a scalar (f) times a vector $\vec{a}$ is $\vec{\nabla}\times(f\vec{a}) =$ _____.

PART – II

2. Answer any *eight* questions : $1\frac{1}{2}\times 8$

(a) Find the solution of the first order linear differential equation

$$\frac{dy}{dx}+P(x)y=Q(x).$$

(*b*) Solve the differential equation

$$x\frac{dy}{dx}+y=x^3+x$$

(*c*) Define Wornskian of a function.

(*d*) Find the first order partial derivatives of

$$u=e^x \sin y.$$

(*e*) Find the unit vector perpendicular to each of the vectors

$$\vec{A}=2\hat{i}-\hat{j}+\hat{k} \text{ and } \vec{B}=3\hat{i}+4\hat{j}-\hat{k}$$

(*f*) Find the angle between the two vectors

$$\left(2\hat{i}+2\hat{j}+3\hat{k}\right) \text{ and } \left(6\hat{i}-3\hat{j}+2\hat{k}\right).$$

(*g*) Find the unit normal to the surface $xy^3z^2=4$ at (–1, –2, 2).

(*h*) Show that

$$x\delta(x)=0.$$

(*i*) Evaluate

$$\int_{x=0}^{1}\int_{y=0}^{2}\left(x^2+3xy^2\right)dxdy.$$

(*j*) Show that position vector $\vec{r}$ is irrotational.

PART – III

3. Answer any *eight* questions : 2 × 8

(*a*) Solve the given differential equation

$$(x^2+y^2)dx-2xydy=0.$$

(*b*) If $u=\log(x^3+y^3+z^3-3xyz)$ then prove that

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=\frac{-9}{(x+y+z)^2}$$

(*c*) Check whether the following functions are linearly independent or not :

$$e^x\cos x,\ e^x\sin x.$$

(*d*) If $u=(y-z)(z-x)(x-y)$ then show that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

(*e*) Show that the Dirac delta function can be represented as limit of Gaussian function.

(*f*) Prove that

$$\vec{A} = 3y^2z^2\hat{i} + 3x^2z^2\hat{j} + 3x^2y^2\hat{k}$$

is a solenoidal vector

(*g*) Derive an expression for velocity in spherical polar coordinates.

(*h*) Prove that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + 2\hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$ are coplanar.

(*i*) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\vec{\nabla} r = \vec{r}/r$.

(*j*) If $u = \dfrac{x+y}{1-xy}, V = \tan^{-1}x + \tan^{-1}y$, find the value of $\dfrac{\partial(u,v)}{\partial(x,y)}$.

PART – IV

Answer **all** questions : 6 × 4

(*Turn Over*)

4. (*a*) Solve the equation

$$\left(8y - x^2y\right)\frac{dy}{dx} + (x - xy^2) = 0.$$ 6

Or

(*b*) If $y_1 = e^{-x}\cos x, y_2 = e^{-x}\sin x$ and

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ then

(*i*) Calculate Wornskian determinant. 3

(*ii*) Verify that y_1 and y_2 satisfy the given differential equation. 3

5. (*a*) Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$. 6

Or

(*b*) Define scalar tripple product. Explain scalar tripple product with its physical significance and features. 6

6. (*a*) Find the curl in terms of orthogonal curvilinear coordinates. 6

Or

(*b*) Derive an expression for velocity and acceleration in cylindrical coordinates. 6

7. (*a*) Find the directional derivative of $\frac{1}{r}$ in the direction $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. 6

Or

(*b*) State Gauss divergence theorem. Evaluate

$$\iint_S \vec{F} \cdot \hat{n} dS$$

where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 6